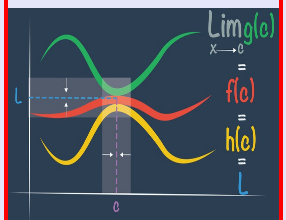
Math 261

Spring 2023

Lecture 16



Prove $\frac{d}{dx}[c] = 0$

$f(x) = c$

$$\frac{d}{dx}[f(x)] = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} \frac{0}{h} = \boxed{0}$$

Prove $\frac{d}{dx}[c\,f(x)] = c\,\frac{d}{dx}[f(x)]$

Let $g(x) = c\,f(x)$

$$\frac{d}{dx}[g(x)] = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{c f(x+h) - c f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c[f(x+h) - f(x)]}{h}$$

$$= c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= c\,\frac{d}{dx}[f(x)]$$

Prove $\frac{d}{dx}\left[x^n\right] = n x^{n-1}$

$f(x) = x^n$

$$\frac{d}{dx}\left[f(x)\right] = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^n} + n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \cdots + h^n - \cancel{x^n}}{h}$$

$$= \lim_{h \to 0} \frac{n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \cdots + h^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(n x^{n-1} + \cancel{\frac{n(n-1)}{2} x^{n-2} h + \cdots + h^{n-1}}\right)}{\cancel{h}}$$

$$= n x^{n-1}$$

ex: $\frac{d}{dx}\left[5x^4\right] = 5 \frac{d}{dx}\left[x^4\right] = 5 \cdot 4 x^{4-1} = \boxed{20x^3}$

$$\frac{d}{dx}\left[-8 \cos x\right] = -8 \frac{d}{dx}\left[\cos x\right] = -8 \cdot (-\sin x)$$

$$= \boxed{8 \sin x}$$

Prove $\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]$

$K(x) = f(x) + g(x)$

$$\frac{d}{dx}\left[K(x)\right] = \lim_{h \to 0} \frac{K(x+h) - K(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h}\right]$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]$$

$$\frac{d}{dx}\left[8 + \sin x\right] = \frac{d}{dx}\left[8\right] + \frac{d}{dx}\left[\sin x\right] = 0 + \cos x = \boxed{\cos x}$$

$$\frac{d}{dx}\left[x^2 + \sqrt{x}\right] = \frac{d}{dx}\left[x^2\right] + \frac{d}{dx}\left[\sqrt{x}\right] \qquad x^{1/2}$$

$$= 2x^{2-1} + \frac{1}{2} x^{\frac{1}{2}-1}$$

$$= 2x^1 + \frac{1}{2} x^{-1/2} = \boxed{2x + \frac{1}{2\sqrt{x}}}$$

Given $f(x) = \frac{1}{3}x^3 - 16x + 10$

1) find $f'(x)$

$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}\left[\frac{1}{3}x^3 - 16x + 10\right]$

$= \frac{d}{dx}\left[\frac{1}{3}x^3\right] - \frac{d}{dx}[16x] + \frac{d}{dx}[10]$

$= \frac{1}{3} \cdot 3x^2 - 16 \cdot 1 + 0$

$= x^2 - 16$

2) Solve $f'(x) = 0$

$x^2 - 16 = 0$

$\vdots$

$\boxed{x = \pm 4}$

Given $f(x) = \frac{1}{2}x - \sin x$

$\frac{1}{2}\frac{d}{dx}[x] = \frac{1}{2} \cdot 1$
$= \frac{1}{2}$

1) find $f'(x)$

$f'(x) = \frac{d}{dx}\left[\frac{1}{2}x - \sin x\right] = \frac{d}{dx}\left[\frac{1}{2}x\right] - \frac{d}{dx}[\sin x]$

$= \boxed{\frac{1}{2} - \cos x}$

2) Solve $f'(x) = 0$ on interval $[0, 2\pi)$.

$\frac{1}{2} - \cos x = 0$ $\quad \cos x = \frac{1}{2}$ $\quad$ QI & QIV

Ref. Angle
$60° = \frac{\pi}{3}$

QI $\rightarrow x = \frac{\pi}{3}$

QIV $\rightarrow x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Prove $\frac{d}{dx}\left[f(x)\cdot g(x)\right] = \frac{d}{dx}\left[f(x)\right]\cdot g(x) + f(x)\cdot\frac{d}{dx}\left[g(x)\right]$

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right] = \lim_{h\to 0}\frac{f(x+h)\cdot g(x+h) - f(x)\cdot g(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0}\left[\frac{g(x+h)\cdot\left[f(x+h)-f(x)\right]}{h} + \frac{f(x)\cdot\left[g(x+h)-g(x)\right]}{h}\right]$$

$$= \lim_{h\to 0}\frac{\overset{g(x)}{\cancel{g(x+h)}}\left[f(x+h)-f(x)\right]}{h} + \lim_{h\to 0}\frac{\overset{f(x)}{f(x)}\cdot\left[g(x+h)-g(x)\right]}{h}$$

$$= g(x)\cdot\frac{d}{dx}\left[f(x)\right] + f(x)\cdot\frac{d}{dx}\left[g(x)\right]$$

$$\frac{d}{dx}\left[x^5\cdot\sin x\right] = \frac{d}{dx}\left[x^5\right]\cdot\sin x + x^5\cdot\frac{d}{dx}\left[\sin x\right]$$

$$= \boxed{5x^4\cdot\sin x + x^5\cdot\cos x}$$

$$\frac{d}{dx}\left[\sin x\cdot\cos x\right] = \frac{d}{dx}\left[\sin x\right]\cdot\cos x + \sin x\cdot\frac{d}{dx}\left[\cos x\right]$$

$$= \cos x\cdot\cos x + \sin x\cdot -\sin x$$

$$= \cos^2 x - \sin^2 x = \boxed{\cos 2x}$$

find $\frac{d}{dx}\left[\sqrt[3]{x}\ \cos x\right]$

Using Product Rule

$$= \frac{d}{dx}\left[\sqrt[3]{x}\right]\cdot\cos x + \sqrt[3]{x}\cdot\frac{d}{dx}\left[\cos x\right]$$

$$= \frac{d}{dx}\left[x^{1/3}\right]\cos x + \sqrt[3]{x}\cdot(-\sin x)$$

Using Power Rule

$$= \frac{1}{3}x^{1/3-1}\cdot\cos x + \sqrt[3]{x}\cdot(-\sin x)$$

$$= \boxed{\frac{1}{3\sqrt[3]{x^2}}\cdot\cos x - \sqrt[3]{x}\cdot\sin x}$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

Find

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{[\cos x]^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\boxed{\frac{d}{dx}[\tan x] = \sec^2 x}$$

Find $f'(x)$ for $f(x) = x\tan x$

$$f'(x) = \frac{d}{dx}[x \tan x]$$

$$= \frac{d}{dx}[x]\cdot \tan x + x \cdot \frac{d}{dx}[\tan x]$$

$$= 1 \cdot \tan x + x \cdot \sec^2 x$$

$$= \boxed{\tan x + x\cdot \sec^2 x}$$